A Note on the Mahalanobis Model

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'THE present note seeks to explore, in a little more detail, some of Professor Shigeto Tsuru's comments, in the Annual Number of the Economic Weekly,' on Professor Mahalanobis' four-sector planning model.- In the discussion, we will treat the model as a purely theoretical framework and ignore its bearing on the country's Second Five Year Plan.

Professor Tsuru finds himself unreconciled to the following statement by Professor Mahalanobis:

'With any given amount of total assets formation (for example, Rs 5,600 crores) and with any given set of values of the B coefficients (for example, those used in this paper) there would be, In principle, an optimum allocation of resources in relation to the basic objectives (such as an increase of employment of 11 million and a long range rate of increase of 5 per cent per year[8] Although the problem, in his opinion, has been correctly set here, Professor Tsuru's qualm is that 'The solving 'as such of a system of simultaneous equations of the kind set up by him (Professor Mahalanobis) is no guarantee that an "optimum" will be struck.'

Is there really an issue of optimisation involved in the problem? An operational model works within certain boundary conditions, and, in addition, a number of other values, such as those of the structural parameters, are also usually given as data. The problem then can be posed in either of two ways. Assuming certain orders of magnitude for the instrument variables, what values can be obtained for the unknowns? Or, in the alternative, what values must the instrument variables assume in order that a number of targets, set a priori, may be reached? Once the problem is finalised, the next step is to fill into the system of equations the values of the knowns together with the values, desired to be realised, of the targets. The solution will then give the values of the unknowns consistent with these. The only necessary condition in order that a unique set of solutions may be obtained is that the number of equations must be equal to the number of unknowns.

The validity of the Mahalanobis model will also depend upon whether there is numerical equivalence between equations and unknowns. In case the condition is satisfied, both Professor Mahalanobis- claim that the solution represents the optimum, and Professor Tsuru's caveat about the claim, will really be irrelevant. The question of optimisation can arise only where a number of alternative solutions are possible. But where specific boundary conditions and parametric values inhere to a model, and there are no degrees of freedom left, we are necessarily headed for a unique solution.

Stated explicitly, Professor Mahalanobis' model consists of the following equations:

$$Yk + Yc_1 + Yc_2 + Yc_3 = Rs\ 2{,}900\ crores \quad (1)$$

$$Ik + Ic_1 + Ic_2 + Ic_3 = Rs\ 5{,}600\ crores \quad (2)$$

$$nk + nc_1 + nc_2 + nc_3 = 11\ million \quad (3)$$

$$Yk = \beta k.Ik \quad (4)$$

$$Yc_1 = \beta_1.Ic_1 \quad (5)$$

$$Yc_2 = \beta_2.Ic_2 \quad (6)$$

$$Yc_3 = \beta_3.Ic_3 \quad (7)$$

$$nc_k = \frac{Ik}{\theta_k} \quad (8)$$

$$nc_1 = \frac{Ic_1}{\theta_1} \quad (9)$$

$$nc_2 = \frac{Ic_2}{\theta_2} \quad (10)$$

$$nc_3 = \frac{Ic_3}{\theta_3} \quad (11)$$

Y's are the stipulated income increases in the four sectors k (basic investment goods), c, (factory consumer goods), c_2 (household industries including agriculture), and c_3 (services); I's are the respective investments; n's represent the allocation of manpower; B's (income coefficients of capital) and O's (capital-labour ratios) are the structural coefficients. The first three equations express the boundary conditions stated in the problem, namely, the creation of additional income amounting to Rs 2,900 crores and additional employment of the order of 11 million on an aggregate new investment of Rs 5,600 crores. The equations (4) to (11) are structural identities.[5]

In Professor Mahalanobis' system, the values of the parameters B' and O's are given. We are thus left with twelve unknowns, namely, the Y's, I's and n's, while the number of equations is only eleven. Clearly, therefore, the system cannot be solved so long as the number of unknowns is not reduced by one. This is done when Professor Mahalanobis also supplies us with the allocation parameter of investment in sector k: one-third of total investment must take place in the investment-goods sector. The equivalence between the number of equations and of unknowns is hence reached, and the system becomes determinate.

Now it can certainly be argued that since the boundary conditions and the given structural coefficients are by themselves insufficient to solve the system, its applicability to the problem formulated by Professor Mahalanobis is open to question. In order to determine the proper 'allocation of resources in relation to the basic objectives' it first becomes necessary to assume a certain allocation for the investment goods sector; the solution therefore

1 Shigeto Tsuru 'Some Theoretical Doubts on India's Plan-frame', Economic Weekly, Annual Number, January 1957, pp 77-79.

P C Mahalanobis, 'The Approach of Operational Research to Planning in India', Sankhya, Vol 16, Parts 1 and 2; specially Chapter 4 and Appendix.

[3] Mahalanobis, op cit p 16.

[4] Tsuru, op cit p 79.

[5] It will be noticed that although Professor Mahalanobis mentions Yo, initial national income, and in, rate of increase of national income, while formulating his problem (ibid, p 34), they do not in fact occur in the model.

begs the question' Besides, as can be shown, any number of alternative solutions can be obtained by varying the allocation parameter of investment for the investment goods sector, and each of them will be consistent with the given values of the structural parameters and satisfy the key objectives of generating Rs 2,900 crores of additional income and 11 million of additional employment with an investment outlay of Rs 5,600 crores. Moreover, the single degree of freedom in the system can be exercised in respect of any of the twelve unknowns, and still we will reach a proper solution. Instead of choosing the order of investment in the investment-goods sector, one could have assumed freely the amount of investment in any of the other three sectors c^1, c_2 and c_3—or the figure of income or of employment for any of the four sectors—the system would still be determinate. Thus viewed, Professor Tsuru's rebuttal of the Mahalanobis claim of 'optimum solution' is indeed valid.

Actually, a solution can be obtained even with a higher value of the allocation parameter of investment for sector k. Thus, on the assumption that 40 per cent of total investment is allotted to the basic investment goods sector, we get the following results in rounded figures:

sectors	investment	increase in income	increase in employment
	Rs crores	Rs crores	million
k	2,240	450	1.1
c_1	450	160	.5
c_2	1,220	1,530	4.9
c_3	1,690	760	4.5
	5,600	2,900	11.0

The three boundary conditions are duly met. Similarly, even where investment in k is taken to be 50 per cent of the aggregate volume, a meaningful solution can still be secured:

sectors	Investment	increase In income	increase In employment
	Rs crores	Rs crores	million
k	2,800	560	1.4
c_1	—	—	—
c_2	1,400	1,750	5.6
c_3	1,400	630	4.8
	5,600	2,940	10.8

Here too, the boundary conditions are almost satisfied; the solution however demands that no investment takes place in the factory consumer goods sector. The eleven unknowns can be solved for yet higher values of I_k but from now on some of the individual values will be negative, and so the solutions will not be interesting from the operational point of view.

Of course, if it is made explicit as Professor Mahalanobis seems to do at another place[6]—that the allocation parameter of investment for sector k=33 per cent is the highest feasible value under Indian conditions, I_k then becomes one of the boundary conditions, and we proceed towards a unique solution. But in no sense can it be claimed that this unique solution will necessarily yield the optimum allocation of resources.

By now, besides, another basic weakness in the model becomes apparent. The sectoral increases in income and employment are the unknowns in the model. Given the values of ß's, $\theta_{'s}$, I_k and the three boundary conditions, these sectoral income and employment increases are obtained as solutions. The model however does not state whether these specific income (therefore, output) increases in the different sectors are consistent with each other. Given the volume of over-all increase in income of Rs 2,900 crores, will the consequent increase in demand in the four sectors be properly matched by output increases of the order of Rs 370 crores, Rs 340 crores, Rs 1,470 crores and Rs 720 crores respectively? As shown in the illustrations above, where one degree of freedom exists, it is always possible to work out a number of alternative solutions each of which will adhere to the basic objectives and to the given values of the structural co efficients. But there is no guarantee that in any of these instances the appropriate balance will be struck between the respective increases in output in the four sectors. This is equally true about Professor Mahalanobis' numerical solution.

In the same way, the expansion of employment in each of the four sectors, adding up to 11 million, is a derived solution, and an element of doubt will persist whether employment increase in these relative orders will be operationally feasible. In fact, it is arguable that the problem of economic planning should not stop- at stipulating merely the over-all targets in respect of employment and income; the question of distribution of these targets over the several sectors, so that the balance between supply and demand is ensured for each, should also be considered and settled at the beginning. That is to say, in the numerical model, it should have been given as datum what sectoral breakdown of the aggregate income target of Rs 2,900 crores would maintain the internal balance, and, also the target of 11 million of additional employment should have been specifically stated in terms of the increases considered feasible in the four sectors.[7]

The strain of the argument above hence is that Y's and n's must be considered as data instead of as unknowns to be solved. Obviously then, if the model is not to be discarded, another set of eight values, which are taken by Professor Mahalanobis as known, must now be regarded as unknowns the orders of magnitude of which have to be obtained as solutions. These categories, we suggest, should be the four ß's and the four θ's.

This is actually a plea for inverting the problem. In the setting out of the problem, instead of starting with given values of the structural coefficients, these can be regarded as parameters to be explored. The numerical equivalence between equations and unknowns is in that case restored, and, despite our treating the Y's and n's as data, the system is no longer over-determined.

For it can well be maintained that,, in the context of planning for an under-developed country, it makes little sense to start with a priori values for the structural coefficients. Since planning concerns developments at the margin, what values

[6] ibid, p 29

[7] Another obvious imbalance in the derived solutions may be noted here. Per capita output in the sectors k, c_1 c_2 and c_3 works out respectively at Rs 4,111, Rs 3,091, Rs 3,127 and Rs 1,674; or, if they are averaged for one year, at Rs 822, Rs 618, Rs 625 and Rs 335 respectively. However, when Professor Mahalanobis splits sector c_3 into (i) agriculture and (ii) small and household enterprises, and solves them with given assumptions for Bs and O,'s, the per capita annual output for agriculture then turns out to be Rs 1,371. Thus the model yields a per capita output, for agriculture which is even higher than that for the basic investment goods sector!

the coefficients should assume is a task for the planners to decide, for these are directly dependent on the technology that is adopted. Both the income coefficient and the capital-labour ratio' are functions of the degree of capital intensity prevalent at any point of time, and it should be given to the planners to decide the appropriate ranges of capital-intensity for the various sectors in the light of available total resources, and taking into consideration the aggregative and sectoral targets of income and employment.

Thus, given the target variables (Y's and n's) and the boundary conditions (including I_k), it will be possible to work out the orders of magnitude of Ic_p Ic_2 and Ic_v and such values of B's and $()$'s as will be consistent with the initial conditions set. The planners can then go ahead with the work of installing the degrees of capital intensity in the different sectors called for by these obtained solutions of the structural coefficients. Where it is discovered that the necessary ranges of capital-intensity are impossible of achievement in a particular sector or sectors because of technical limitations,[9] it should be possible to go back to the target variables and make proper adjustments in them, without necessarily disturbing the over-all targets.[10]

On the other hand, a distinct hazard exists if one starts with given values of the structural coefficients, obtains certain allocation parameters of investment consistent with these values, and proceeds to make the sectoral investments accordingly. For if the B's and θ's turn out in actuality to be different from the assumptions, the income and employment targets cannot be reached; since the investment operations would already have been started, there might be very little scope for retrieving the situation. Professor Mahalanobis' assertion, namely, that if the values of the B coefficients turn out in practice to be somewhat different then the solution used in the plan [that is, in his numerical model would not be a true optimum but may still serve as quite an efficient solution[11] therefore baffles explanation. Clearly, if B's are different, the income targets cannot be attained; there will no doubt be a solution to the system, but not one which will fulfil 'the basic objectives' initially set

" As Professor Tsuru shows, they are inter-related, ibid, p 78.

" Thus, in the investment goods sector, there may be only a narrow limit within which k can move.

To The possibility of second-order adjustments between, for example, c_1 and c_2 may often exist.

" ibid, p 46